Separable Equations: #7 – 23 odd and 24.

In Problems 7–16, solve the equation.

7. $\dfrac{dy}{dx} = \dfrac{1 - x^2}{y^2}$

8. $\dfrac{dy}{dx} = \dfrac{1}{xy^3}$

9. $\dfrac{dy}{dx} = y(2 + \sin x)$

10. $\dfrac{dx}{dt} = 3xt^2$

11. $\dfrac{dy}{dx} = \dfrac{\sec^2 y}{1 + x^2}$

12. $x\dfrac{dv}{dx} = \dfrac{1 - 4v^2}{3v}$

13. $\dfrac{dx}{dt} + x^2 = x$

14. $\dfrac{dy}{dx} = 3x^2(1 + y^2)$

15. $y^{-1}\,dy + ye^{\cos x}\sin x\,dx = 0$

16. $(x + xy^2)\,dx + e^{x^2}y\,dy = 0$

In Problems 17–26, solve the initial value problem.

17. $y' = x^3(1 - y)\ , \qquad y(0) = 3$

18. $\dfrac{dy}{dx} = (1 + y^2)\tan x\ , \qquad y(0) = \sqrt{3}$

19. $\dfrac{dy}{d\theta} = y\sin\theta\ , \qquad y(\pi) = -3$

20. $\dfrac{dy}{dx} = \dfrac{3x^2 + 4x + 2}{2y + 1}\ , \qquad y(0) = -1$

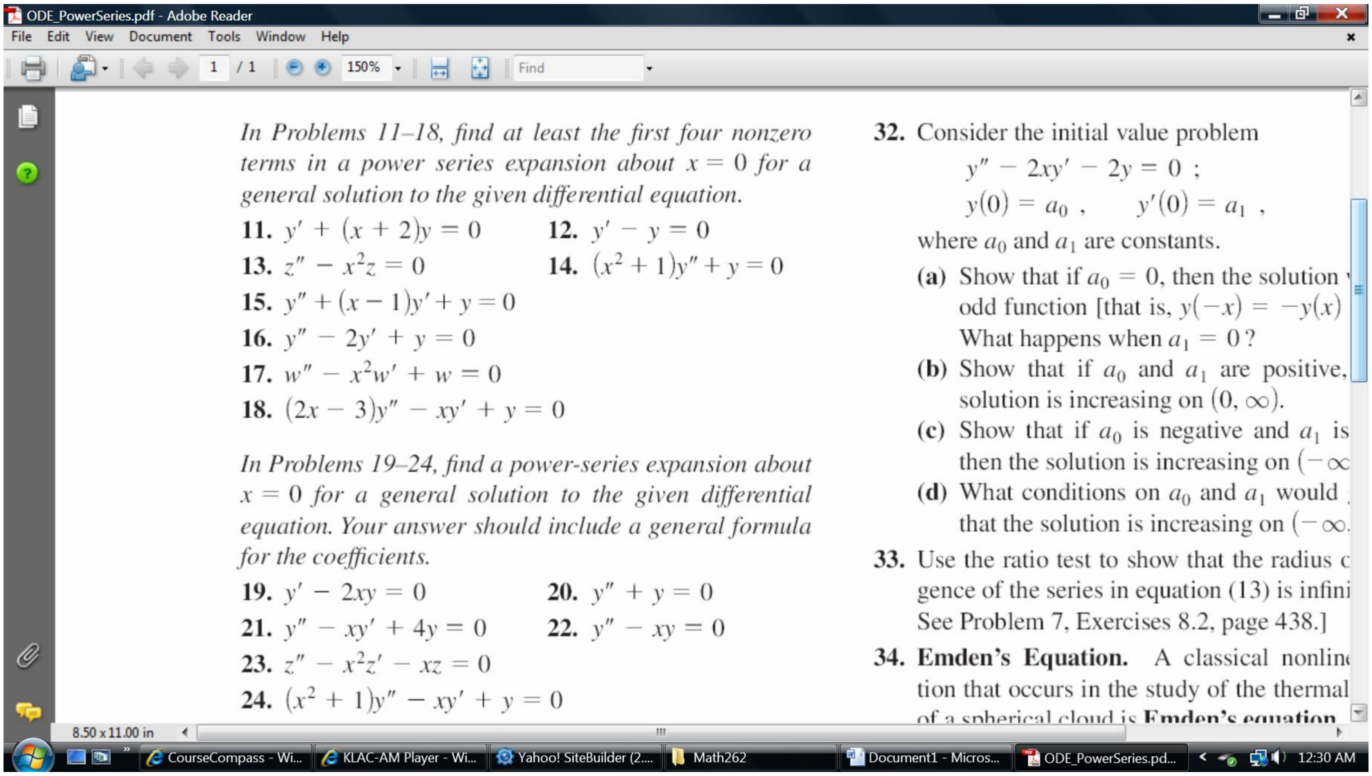
21. $\dfrac{dy}{dx} = 2\sqrt{y + 1}\cos x\ , \qquad y(\pi) = 0$

22. $x^2\,dx + 2y\,dy = 0\ , \qquad y(0) = 2$

23. $\dfrac{dy}{dx} = 2x\cos^2 y\ , \qquad y(0) = \pi/4$

24. $\dfrac{dy}{dx} = 8x^3e^{-2y}\ , \qquad y(1) = 0$

Power Series Solution: # 11, 17, 19, 27



In Problems 11–18, find at least the first four nonzero terms in a power series expansion about $x = 0$ for a general solution to the given differential equation.

11. $y' + (x + 2)y = 0$

12. $y' - y = 0$

13. $z'' - x^2 z = 0$

14. $(x^2 + 1)y'' + y = 0$

15. $y'' + (x - 1)y' + y = 0$

16. $y'' - 2y' + y = 0$

17. $w'' - x^2 w' + w = 0$

18. $(2x - 3)y'' - xy' + y = 0$

In Problems 19–24, find a power-series expansion about $x = 0$ for a general solution to the given differential equation. Your answer should include a general formula for the coefficients.

19. $y' - 2xy = 0$

20. $y'' + y = 0$

21. $y'' - xy' + 4y = 0$

22. $y'' - xy = 0$

23. $z'' - x^2 z' - xz = 0$

24. $(x^2 + 1)y'' - xy' + y = 0$

32. Consider the initial value problem

$$y'' - 2xy' - 2y = 0 \ ;$$
$$y(0) = a_0 \ , \qquad y'(0) = a_1 \ ,$$

where a_0 and a_1 are constants.

(a) Show that if $a_0 = 0$, then the solution
odd function [that is, $y(-x) = -y(x)$
What happens when $a_1 = 0$?

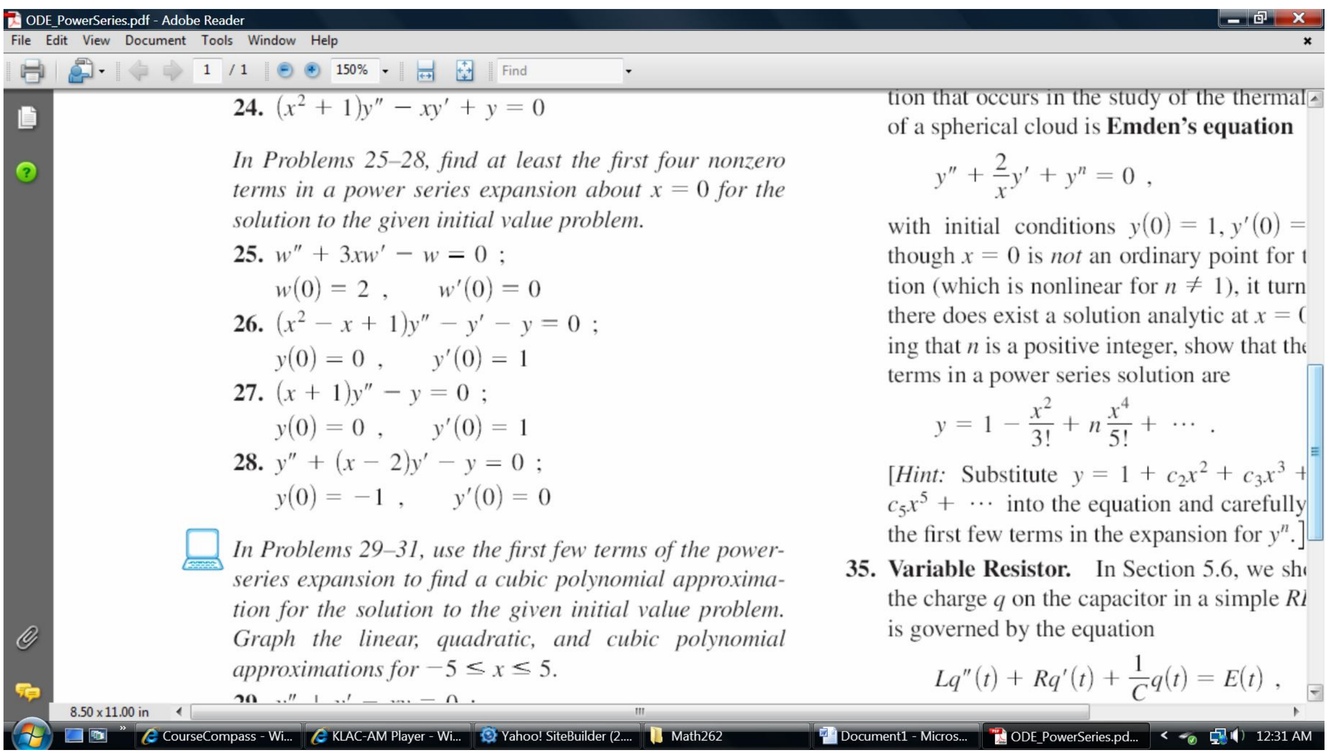
(b) Show that if a_0 and a_1 are positive,
solution is increasing on $(0, \infty)$.

(c) Show that if a_0 is negative and a_1 is
then the solution is increasing on $(-\infty$

(d) What conditions on a_0 and a_1 would
that the solution is increasing on $(-\infty$.

33. Use the ratio test to show that the radius c
gence of the series in equation (13) is infini
See Problem 7, Exercises 8.2, page 438.]

34. Emden's Equation. A classical nonlin
tion that occurs in the study of the thermal



24. $(x^2 + 1)y'' - xy' + y = 0$

In Problems 25–28, find at least the first four nonzero terms in a power series expansion about $x = 0$ for the solution to the given initial value problem.

25. $w'' + 3xw' - w = 0$;
$w(0) = 2$, $w'(0) = 0$

26. $(x^2 - x + 1)y'' - y' - y = 0$;
$y(0) = 0$, $y'(0) = 1$

27. $(x + 1)y'' - y = 0$;
$y(0) = 0$, $y'(0) = 1$

28. $y'' + (x - 2)y' - y = 0$;
$y(0) = -1$, $y'(0) = 0$

In Problems 29–31, use the first few terms of the power-series expansion to find a cubic polynomial approximation for the solution to the given initial value problem. Graph the linear, quadratic, and cubic polynomial approximations for $-5 \le x \le 5$.

tion that occurs in the study of the thermal
of a spherical cloud is **Emden's equation**

$$y'' + \frac{2}{x}y' + y^n = 0 \ ,$$

with initial conditions $y(0) = 1$, $y'(0) =$
though $x = 0$ is *not* an ordinary point for t
tion (which is nonlinear for $n \neq 1$), it turn
there does exist a solution analytic at $x = ($
ing that n is a positive integer, show that th
terms in a power series solution are

$$y = 1 - \frac{x^2}{3!} + n\frac{x^4}{5!} + \cdots \ .$$

[*Hint:* Substitute $y = 1 + c_2x^2 + c_3x^3 +$
$c_5x^5 + \cdots$ into the equation and carefully
the first few terms in the expansion for y^n.]

35. Variable Resistor. In Section 5.6, we sh
the charge q on the capacitor in a simple RI
is governed by the equation

$$Lq''(t) + Rq'(t) + \frac{1}{C}q(t) = E(t) \ ,$$