

Separable Equations: #7 – 23 odd and 24.

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In Problems 7–16, solve the equation.

7.  $\frac{dy}{dx} = \frac{1 - x^2}{y^2}$       8.  $\frac{dy}{dx} = \frac{1}{xy^3}$

9.  $\frac{dy}{dx} = y(2 + \sin x)$       10.  $\frac{dx}{dt} = 3xt^2$

11.  $\frac{dy}{dx} = \frac{\sec^2 y}{1 + x^2}$       12.  $x \frac{dv}{dx} = \frac{1 - 4v^2}{3v}$

13.  $\frac{dx}{dt} + x^2 = x$       14.  $\frac{dy}{dx} = 3x^2(1 + y^2)$

15.  $y^{-1} dy + ye^{\cos x} \sin x dx = 0$

16.  $(x + xy^2)dx + e^{x^2}y dy = 0$

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solving a c  
use definite  
limit). For c  
 $\frac{dy}{dx} =$   
The differ  
and multiplic  
tion from x  
 $\int_{x=2}^{x=x_1}$

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In Problems 17–26, solve the initial value problem.

17.  $y' = x^3(1 - y)$ ,  $y(0) = 3$

18.  $\frac{dy}{dx} = (1 + y^2)\tan x$ ,  $y(0) = \sqrt{3}$

19.  $\frac{dy}{d\theta} = y \sin \theta$ ,  $y(\pi) = -3$

20.  $\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2y + 1}$ ,  $y(0) = -1$

21.  $\frac{dy}{dx} = 2\sqrt{y + 1} \cos x$ ,  $y(\pi) = 0$

22.  $x^2 dx + 2y dy = 0$ ,  $y(0) = 2$

23.  $\frac{dy}{dx} = 2x \cos^2 y$ ,  $y(0) = \pi/4$

24.  $\frac{dy}{dx} = 8x^3 e^{-2y}$ ,  $y(1) = 0$

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If we let  $t$  be the variabl  
 $x_1$  by  $x$  and  $y(2)$  by 1, th  
tion to the initial value p  
 $y(x) = \left(1 - \int_2^x e^t$   
Use definite integration t  
the initial value problem:  
(a)  $dy/dx = e^{x^2}$ ,  $y$   
(b)  $dy/dx = e^{x^2}y^{-2}$ ,  
(c)  $dy/dx = \sqrt{1 + \sin}$   
(d) Use a numerical inte  
Simpson's rule, des  
approximate the solu  
three decimal places

Power Series Solution: # 11, 17, 19, 27

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In Problems 11–18, find at least the first four nonzero terms in a power series expansion about  $x = 0$  for a general solution to the given differential equation.

11.  $y' + (x + 2)y = 0$       12.  $y' - y = 0$   
 13.  $z'' - x^2z = 0$       14.  $(x^2 + 1)y'' + y = 0$   
 15.  $y'' + (x - 1)y' + y = 0$   
 16.  $y'' - 2y' + y = 0$   
 17.  $w'' - x^2w' + w = 0$   
 18.  $(2x - 3)y'' - xy' + y = 0$

In Problems 19–24, find a power-series expansion about  $x = 0$  for a general solution to the given differential equation. Your answer should include a general formula for the coefficients.

19.  $y' - 2xy = 0$       20.  $y'' + y = 0$   
 21.  $y'' - xy' + 4y = 0$       22.  $y'' - xy = 0$   
 23.  $z'' - x^2z' - xz = 0$   
 24.  $(x^2 + 1)y'' - xy' + y = 0$

32. Consider the initial value problem  
 $y'' - 2xy' - 2y = 0$  ;  
 $y(0) = a_0$  ,  $y'(0) = a_1$  ,  
 where  $a_0$  and  $a_1$  are constants.  
 (a) Show that if  $a_0 = 0$ , then the solution is an odd function [that is,  $y(-x) = -y(x)$ ]. What happens when  $a_1 = 0$  ?  
 (b) Show that if  $a_0$  and  $a_1$  are positive, the solution is increasing on  $(0, \infty)$ .  
 (c) Show that if  $a_0$  is negative and  $a_1$  is positive, then the solution is increasing on  $(-\infty, 0)$ .  
 (d) What conditions on  $a_0$  and  $a_1$  would guarantee that the solution is increasing on  $(-\infty, \infty)$ ?

33. Use the ratio test to show that the radius of convergence of the series in equation (13) is infinite. [See Problem 7, Exercises 8.2, page 438.]

34. **Emden's Equation.** A classical nonlinear problem that occurs in the study of the thermal structure of a spherical cloud is **Emden's equation**

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24.  $(x^2 + 1)y'' - xy' + y = 0$

In Problems 25–28, find at least the first four nonzero terms in a power series expansion about  $x = 0$  for the solution to the given initial value problem.

25.  $w'' + 3xw' - w = 0$  ;  
 $w(0) = 2$  ,  $w'(0) = 0$   
 26.  $(x^2 - x + 1)y'' - y' - y = 0$  ;  
 $y(0) = 0$  ,  $y'(0) = 1$   
 27.  $(x + 1)y'' - y = 0$  ;  
 $y(0) = 0$  ,  $y'(0) = 1$   
 28.  $y'' + (x - 2)y' - y = 0$  ;  
 $y(0) = -1$  ,  $y'(0) = 0$

In Problems 29–31, use the first few terms of the power-series expansion to find a cubic polynomial approximation for the solution to the given initial value problem. Graph the linear, quadratic, and cubic polynomial approximations for  $-5 \leq x \leq 5$ .

29.  $y'' + y = 0$  ;  $y(0) = 1$  ,  $y'(0) = 0$

tion that occurs in the study of the thermal structure of a spherical cloud is **Emden's equation**

$$y'' + \frac{2}{x}y' + y^n = 0$$

with initial conditions  $y(0) = 1$ ,  $y'(0) = 0$ . Although  $x = 0$  is not an ordinary point for this equation (which is nonlinear for  $n \neq 1$ ), it turns out that there does exist a solution analytic at  $x = 0$ . Assuming that  $n$  is a positive integer, show that the first few terms in a power series solution are

$$y = 1 - \frac{x^2}{3!} + n \frac{x^4}{5!} + \dots$$

[Hint: Substitute  $y = 1 + c_2x^2 + c_3x^3 + c_5x^5 + \dots$  into the equation and carefully match the first few terms in the expansion for  $y^n$ .]

35. **Variable Resistor.** In Section 5.6, we showed that the charge  $q$  on the capacitor in a simple RC circuit is governed by the equation

$$Lq''(t) + Rq'(t) + \frac{1}{C}q(t) = E(t)$$