Chapter 9

Lecture 4

Section: 9.4

We have studied inferences about the means of two independent populations. We defined two samples to be *independent* if the sample values selected from one population are not related to, paired or matched with the sample values from the other population. We will now study dependent samples, which we refer to as *matched pairs.*

With matched pairs, there is some relationship so that each value in one sample is paired with a corresponding value in the other sample.

Examples:

1. When conducting an experiment to test the effectiveness of a low-fat diet, the weight of each subject is measured once before the diet and once after the diet.

2. The accuracy of reported weights is analyzed with a sample of people when, for each person, the reported weight is recorded and the actual weight is measured.

Assumptions:

1. The sample data consist of matched pairs.
2. The samples are simple random samples.
3. Either or both of these conditions is satisfied:
   i. The number of matched pairs of sample data is large, $n > 30$
   ii. The pairs of values have differences that are from a population having a distribution that is approximately normal.

Notation or Matched Pairs:

d = individual difference between the two values in a single matched pair all matched pairs.

μ_d = mean value of the differences d for the population of all matched pairs.

$\bar{d}$ = mean value of the differences d for the paired sample data. Equal to the mean of the x – y values.

s_d = standard deviation of the differences d for the paired sample data

n = number of pairs of data

Confidence Intervals Estimate for Matched Pairs:

$$\bar{d} - E < \mu_d < \bar{d} + E$$

$$\text{where } E = t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

df = n – 1

Test Statistic for Matched Pairs:

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}$$

Where H_0: $\mu_d = 0$

df = n – 1

3. 6 people were randomly selected and asked their hourly wage. Then they were asked to show proof of their hourly wage. Suppose the samples are taken from a *normal* population, given below. Test the claim that there is a difference between reported and verified hourly wages at $\alpha = 0.01$.

Reported:	12	10	15	17	35	20
Verified:	11	10	13	16	25	18

4. Construct a 98% confidence interval for the difference in the two populations for problem #1.

5. A random sample of 10 patients is given some new brand of prescription drug to reduce blood pressure. The samples are taken from a *normal* population. Test the claim that the new prescription drug is reducing the blood pressure significantly at a 0.05 level.

Before:	130	122	125	127	133	123	110	105	145	140
After:	120	115	128	123	107	113	118	110	125	122

6. Construct a 90% confidence interval for the difference in the two populations for problem #5.

7. Listed below are heights, in inches, of male statistic students. Is there sufficient evidence to support the claim that male students exaggerate by reporting heights that are greater than their actual measured heights? Use a 0.025 level of significance.

Reported:	64	63	64	65	64	64	63	59	66	64
Actual:	63.5	63.1	63.8	63.4	62.1	64.4	62.7	59.3	65.4	62.2

8. Construct a 95% confidence interval for the difference in the two populations for problem #7.

Paired T-Test and CI: Reported, Measured

Paired T for Reported - Measured

	N	Mean	StDev	SE Mean
Reported	10	63.6000	1.8379	0.5812
Measured	10	62.9900	1.6333	0.5165
Difference	10	0.610000	0.862103	0.272621

97.5% lower bound for mean difference: -0.006712
T-Test of mean difference = 0 (vs > 0): T-Value = 2.24 P-Value = 0.026

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Paired T for Reported - Measured

	N	Mean	StDev	SE Mean
Reported	10	63.6000	1.8379	0.5812
Measured	10	62.9900	1.6333	0.5165
Difference	10	0.610000	0.862103	0.272621

95% CI for mean difference: (-0.006712, 1.226712)
T-Test of mean difference = 0 (vs not = 0): T-Value = 2.24 P-Value = 0.052

9. A listening test was administered to English teachers before and after an institute designed to improve English listening skills.
The maximum possible score on the test was 36:

Sub	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Pre	30	28	31	26	20	30	34	15	28	20	30	29	31	29	34	20	26	25	31	29
Post	29	30	32	30	16	25	31	18	33	25	32	28	34	32	22	7	28	29	32	32

Determine if the institute improved listening skills at the 0.5% significance level.

10. Determine if a 99% confidence interval will give the same conclusion as the hypothesis test above.