

Chapter 3

Lecture 3

Sections 3.4 – 3.5

Measure of Position


We would like to compare values from different data sets.

We will introduce a “z-score” or “standard score”. This measures how many standard deviation from the mean a given number x is. We use the following:

$$z = \frac{x - \bar{x}}{s} \quad \text{or} \quad z = \frac{x - \mu}{\sigma}$$

If the value of x is smaller than the mean, then z will be negative.

At UCLA in a specific quarter, I took two classes that were graded on a curve. In Math, the class had a mean of 80 and standard deviation of 11. In Economics, the class mean was 46 with a standard deviation of 5. I received a grade 90 in Math and a grade of 54 in Economics. Does my grade in Math among the class exceed my grade in Econ among the class?



$$\text{Math Standard Score } z = \frac{90 - 80}{11} = 0.91$$

$$\text{Econ Standard Score } z = \frac{54 - 46}{5} = 1.6$$

This means that my score in my Economics class is relatively higher when compared to the class than that of my Math class.

How many standard deviation is a score of 30 in the economics class?

- *Ordinary Values:* $-2 \leq z\text{-score} \leq 2$,
- *Unusual Values:* $z\text{-score} < -2$ or $z\text{-score} > 2$



Example:

Men have heights with a mean of 69.0in. and a standard deviation of 2.8in.; women have heights with a mean of 63.6 with a standard deviation of 2.5in. If a man is 74in. tall and a women is 70in. tall, who is relatively taller?

Percentiles

Recall that the *median* of 56, 66, 70, 77, 80, 86, 99 was 77.

This means is that 50% of the values are equal to or less than the *median* and 50% of the values are equal to or greater than the *median*. In other words, it separates the top 50% from the bottom 50%.

We are also able to find other values that separate data.

After arranging the data in increasing order:

- Q_1 =First Quartile: Separates the bottom 25% from the top 75%.
This is the same as the P_{25} =25th Percentile.
- Q_2 =Second Quartile: Same as the median.
This is the same as the P_{50} =50th Percentile.
- Q_3 =Third Quartile: Separates the bottom 75% from the top 25%.
This is the same as the P_{75} =75th Percentile.

There are other ways to separate the data.

- P_1 =First Percentile: Separates the bottom 1% from the top 99%.
- P_{10} =Tenth Percentile: Separates the bottom 10% from the top 90%.
This is the same as the D_1 =1st Decile.
- P_{20} =20th Percentile: Separates the bottom 20% from the top 80%.
This is the same as the D_2 =2nd Decile.
- P_{66} =66th Percentile: Separates the bottom 66% from the top 34%.
- P_{95} =95th Percentile: Separates the bottom 95% from the top 5%.
- P_k = k th Percentile: Separates the bottom k % from the top $(100-k)$ %.
This is the general form of percentiles.

Just to name a few.

Finding Percentiles

- Finding a percentile that corresponds to a particular value “ x ” of the data set is as follows:

$$\text{Percentile of } x = \frac{\# \text{ of values less than } x}{\text{total \# of values}} \cdot 100$$

- *Example: The following data represents the final 50 percentages of last semesters Algebra class arranged in increasing order.*

10 13 13 15 16 22 23 24 32 35 36 36 38 40 41 42 43 44
46 48 51 52 53 53 56 57 57 57 58 60 60 62 65 67 71 73
74 76 80 82 86 88 93 94 94 94 96 98 99 99

10 13 13 15 16 22 23 24 32 35 36 36 38 40 41 42 43
44 46 48 51 52 53 53 56 57 57 57 58 60 60 62 65 67
71 73 74 76 80 82 86 88 93 94 94 94 96 98 99 99

- *Find the percentile that corresponds to the value of 22.*

$$\text{Percentile of } 22 = \frac{5}{50} \cdot 100 = 10$$

This tells us that 22 is the 10th percentile ($P_{10} = D_1$). We conclude that 10% of the students are below or equal to 22 and 90% of the class is above or equal to .

- *If a student received a score of 78, what percentile does the student fall in?*

$$\text{Percentile of } 78 = \frac{38}{50} \cdot 100 = 76$$

This tells us that 78 is the 76th percentile (P_{76}). We conclude that 76% of the students are below or equal to 78 and 24% of the class is above or equal to 78.

10 13 13 15 16 22 23 24 32 35 36 36 38 40 41 42 43
 44 46 48 51 52 53 53 56 57 57 57 58 60 60 62 65 67
 71 73 74 76 80 82 86 88 93 94 94 94 96 98 99 99

- Lets find the value of the 70th percentile (P_{70}). We will need to use the following formula.

$$L = \frac{k}{100} \cdot n$$

Where k is the *percentile*, n is the *total # of values*, and L is the Locator that tells us where the value we are looking for is.

$$L = \frac{70}{100} \cdot 50 = 35$$

Since $L =$ is a whole number, what we have to do is get the 35th value and the 36th value and compute their average.

$$P_{70} = D_7 = \frac{71+73}{2} = 72$$

- *If L is a whole number, you must get that number and the number that comes after it, then compute their average.*
- *If L is a decimal number, round up and with that number you will find P_k .*
- *Remember, you will have to order the data in increasing order first.*

Example: Find the 3rd quartile of the data.

20 45 50 54 55 61 63 66 67

$$L = \frac{75}{100} \cdot 9 = 6.75 \approx 7$$

$$Q_3 = P_{75} = 63$$

Statistic defined by using Quartiles.

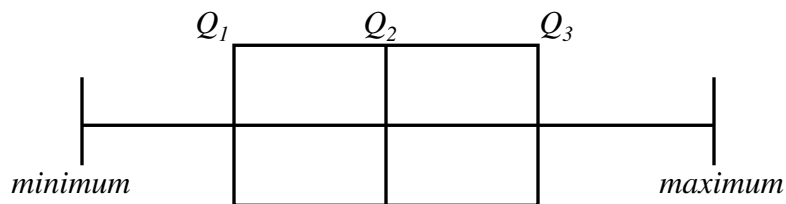
■ Interquartile Range (IQR):

$$Q_3 - Q_1$$

Graphs using Percentiles.

■ Boxplot:

Consists of a 5 number summary that is made up of the *minimum*, Q_1 , Q_2 , Q_3 , and the *maximum*.



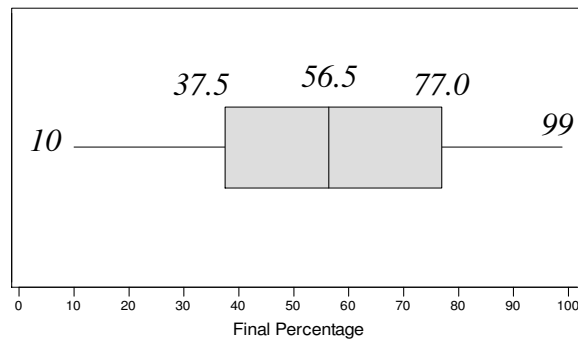
Minitab Printout:

Descriptive Statistics: Final Percentage

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Final %	50	56.44	56.50	56.59	26.02	3.68

Variable	Minimum	Maximum	Q1	Q3
Final %	10.00	99.00	37.50	77.00

Boxplot of Final Percentage



Example:

In “Age of Oscar winning Best Actors and Actresses” by Richard Brown, the author compares the ages of actors and actresses at the time that they won their Oscar. The results for winners from both categories are listed bellow. Use a boxplot to compare their ages.

Male: 32 37 36 32 51 53 33 61 35 45 55 39 76 37
42 40 32 60 38 56 48 48 40 43 62 43 42 44
41 56 39 46 31 47 45 60 46 40 36

Female: 50 44 35 80 26 28 41 21 61 38 49 33 74 30
33 41 31 35 41 42 37 26 34 34 35 26 61 60
34 24 30 37 31 27 39 34 26 25 33